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Singular Perturbation Analysis of Optimal Glide

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I. Introduction

MAXIMUM-RANGE trajectories in the vertical plane are extremely important for gliders that possess a limited amount of initial energy. Also, every airplane becomes a glider if the engine is turned off on purpose or if the engine fails because of fuel shortage or technical reasons. When speeds were relatively low, performance optimization of aircraft was studied on a steady-state basis.[1] The best range glide in a steady-state situation is well known. However, such a result ignores the boundary conditions of the problem. With the development of high-performance aircraft and air-launched gliding vehicles, the dynamic effects could no longer be neglected.

Singular perturbation theory (SPT) has been used extensively for nearly three decades in solving various optimal trajectory problems, including optimal performance and optimal interceptions.[2,3] The basic idea of the SPT is to timescale decouple the dynamic equations into lower-order sets. The advantage of the method is that it can produce analytical closed-loop control laws with minimal sacrifice of accuracy.[4]

All papers that dealt with vertical or three-dimensional flight chose the path angle as the fastest variable with altitude being comparable or slower.[5] The velocity was replaced by the energy, which was always slower than the altitude. It should be emphasized,[5] however, that the only justification for this approximation is that, as verified by numerical experiments, these cases exhibit slow energy changes with relatively faster constant-energy altitude-velocity changes.

The main theme of the present paper is that the timescale of the specific energy is not always necessarily slow relative to the altitude timescale. In particular, gliders in transitions, to and from steady-state glides, change their speed (and accordingly the specific energy) in a relatively fast process, and therefore the classical separation does not hold. Therefore, it makes sense to consider the velocity as a fast variable. (A similar idea is suggested in Ref. 6 for the fast velocity changes during transition to and from steady-state turn.) This is the approach pursued in this work. The flight-path angle is assumed to be faster than the velocity. Sheu et al.[7] assume the energy-altitude separation and derives a SPT solution to the optimal glide problem for maximum range. Two numerical cases are analyzed and compared to the SPT solution. One case shows good agreement, whereas the other does not. This suggests that the suitability of a timescale separation can vary from case to case. The classical widely used steady-state optimal glide cannot be reproduced as the reduced-order solution under the energy-altitude separation. In the present paper, under the just-mentioned assumptions, analytical closed-loop control law and open-loop solution for the maximum range glider trajectory are constructed.

II. Problem Formulation

The dynamical model for the glider is based on the assumption of vertical plane motion in a fixed and flat Earth. Further, the rigid-body dynamics and structural vibrations are ignored. The model is described by the standard point-mass equations of motion,[6] where the range x, altitude h, velocity V, and flight-path angle γ are the state variables (see Sec. III).

The aerodynamic forces are

$$D = C_D \cdot q \cdot s, \qquad L = C_L \cdot q \cdot s, \qquad q = 0.5 \cdot \rho \cdot V^2$$
$$C_D = C_{D_0} + K \cdot C_L^2, \qquad \rho(h) = \rho_{\mathrm{sl}} \cdot \exp(-h/h_{\mathrm{ref}}) \tag{1}$$

The framework of the optimization is that of optimal control, where the cost is the final range and the control is the lift coefficient C_L. The initial boundary conditions include the full-state vector (x_0, h_0, v_0, γ_0), and the only final condition is the altitude h_f.

III. Problem Analysis

In this study the airspeed is assumed to be faster than the altitude, and the path angle γ is considered as the fastest variable. The range and altitude are the slower variables with the same timescale. Thus, the problem is treated with three different timescales. The outer section deals with the quasi-steady flight, where the range and altitude are the state variables, the speed is the actual control, and γ is determined by the other variables. The first boundary-layer formulation regards the slow variables as constant, the speed as the state variable, and γ as the control. The second boundary-layer formulation considers the slow variables and the speed as constants, γ as the state variable, and C_L as an unbounded control. The final time is not specified.

The state equations in the SPT approximation are

$$\frac{\mathrm{d}x}{\mathrm{d}t} = V \cdot \cos(\gamma) \tag{2a}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = V \cdot \sin(\gamma) \tag{2b}$$

$$\varepsilon \cdot \frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{D}{m} - g \cdot \sin(\gamma) \tag{2c}$$

$$\varepsilon^2 \cdot \frac{\mathrm{d}\gamma}{\mathrm{d}t} = \left[\frac{L}{W} - \cos(\gamma)\right] \cdot \frac{g}{V} \tag{2d}$$

The cost function is $J = -x_f$, and the Hamiltonian is

$$\begin{aligned} H = {} & \lambda_x \cdot V \cdot \cos(\gamma) + \lambda_h \cdot V \cdot \sin(\gamma) \\ & + \lambda_v \cdot [-(D/m) - g \cdot \sin(\gamma)] \\ & + \lambda_\gamma \cdot [L/W - \cos(\gamma)] \cdot g/V \end{aligned} \tag{3}$$

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The costates satisfy the Euler–Lagrange equations:

$$\frac{\mathrm{d}\lambda_x}{\mathrm{d}t} = -\frac{\partial H}{\partial x} = 0 \tag{4a}$$

$$\frac{\mathrm{d}\lambda_h}{\mathrm{d}t} = -\frac{\partial H}{\partial h} \tag{4b}$$

$$\varepsilon \cdot \frac{\mathrm{d}\lambda_v}{\mathrm{d}t} = -\frac{\partial H}{\partial V} \tag{4c}$$

$$\varepsilon^2 \cdot \frac{\mathrm{d}\lambda_\gamma}{\mathrm{d}t} = -\frac{\partial H}{\partial \gamma} \tag{4d}$$

Initial and final values of the states and costates are

$$x(0) = 0, \qquad x(t_f)\text{free}, \qquad \lambda_x = -1 \tag{5a}$$

$$h(0) = h_0, \qquad h(t_f) = h_f, \qquad \lambda_h(t_f)\text{free} \tag{5b}$$

$$V(0) = V_0, \qquad V(t_f)\text{free}, \qquad \lambda_v(t_f) = 0 \tag{5c}$$

$$\gamma(0) = \gamma_0, \qquad \gamma(t_f)\text{free}, \qquad \lambda_\gamma(t_f) = 0 \tag{5d}$$

IV. SPT Solution

A. Outer Solution

The outer solution is derived by substituting $\varepsilon = 0$ into the general formalism. The cost function remains unchanged, and because H^0 does not depend explicitly on time, we get

$$H^0 = \lambda_x \cdot V \cdot \cos(\gamma) + \lambda_h \cdot V \cdot \sin(\gamma) = 0 \tag{6}$$

The left-hand sides of Eqs. (4c) and (4d) also become zero, complying with the necessary optimal condition for the controls v and γ:

$$\frac{\partial H}{\partial V} = \frac{\partial H}{\partial \gamma} = \frac{\partial H}{\partial C_L} = 0 \tag{7}$$

From Eqs. (6) and (7) it can be shown that

$$\tan(\gamma^o) = -2\sqrt{C_{D_0} \cdot K}; \qquad C_L^o = \sqrt{C_{D_0}/K} \tag{8}$$

$$\lambda_h^o = -\left(1\big/2\sqrt{C_{D_0} \cdot K}\right); \qquad \lambda_v^o = -\left[v\big/\left(g \cdot 2\sqrt{C_{D_0} \cdot K}\right)\right]$$
$$\lambda_\gamma^o = -(V^2/g) \tag{9}$$

The lift coefficient is identical to the classical coefficient for a quasi-steady-state optimal glide. We can also get explicit forms for the time history of the states.

The outer solution fulfills the slow variable initial and final conditions but not those of the fast variables. The initial boundary layers should complete this requirement for the initial boundary conditions, and the terminal boundary layer should do the same for the final boundary conditions. In the sequel only the initial boundary layer is solved because the terminal one is identical with a reversed time direction. Therefore only the initial boundary conditions are imposed later on.

B. First Inner Boundary Layer

The first inner boundary layer is derived by changing the timescale from t to $\tau_1 = t/\varepsilon$ and then substituting $\varepsilon = 0$ into the general formalism. All of the variables in the first boundary layer should be labeled with an upper index i_1. Also, the slow variables are constant and equal to their initial values. The velocity changes according to

$$\frac{\mathrm{d}V^{i_1}}{\mathrm{d}\tau_1} = -\frac{D^{i_1}}{m} - g \cdot \sin(\gamma^{i_1}) \tag{10}$$

with an initial condition V_0 and a final condition that coincides with the initial outer velocity V_0^o.

The lift coefficient $C_L^{i_1}$ reflects the trim value required to retain the vehicle in longitudinal equilibrium defined by γ^{i_1} and $q^{i_1} = q(h_0, V^{i_1})$. The fastest variable γ obeys an algebraic equation, which enables it to reach instantaneously its optimal required value and basically act as the control of V.

Because the Hamiltonian does not depend explicitly on time, we get

$$H = \lambda_{x_0} \cdot V^{i_1} \cdot \cos(\gamma^{i_1}) + \lambda_{h_0} \cdot V^{i_1} \cdot \sin(\gamma^{i_1})$$
$$+ \lambda_v^{i_1} \cdot \left\{-\left[D\left(h_0, V^{i_1}, \gamma^{i_1}\right)\big/m\right] - g \cdot \sin(\gamma^{i_1})\right\} = 0 \tag{11}$$

The costate equation of λ_γ expresses the fact that γ^{i_1} is the active control in this layer

$$\varepsilon \frac{\mathrm{d}\lambda_y^{i_1}}{\mathrm{d}\tau_1} = -\frac{\partial H}{\partial \gamma^{i_1}} = -\left[-\lambda_{x_0} \cdot V^{i_1} \cdot \sin(\gamma^{i_1}) + \lambda_{h_0} \cdot V \cdot \cos(\gamma^{i_1})\right.$$
$$\left. - \frac{\lambda_v^{i_1}}{m} \cdot \frac{\partial D\left(h_0, V^{i_1}, \gamma^{i_1}\right)}{\partial \gamma^{i_1}} - \lambda_v^{i_1} \cdot g \cdot \cos(\gamma^{i_1})\right] = 0 \tag{12}$$

An implicit expression for $\gamma^{i_1}(q^{i_1})$ is obtained from Eqs. (11) and (12) under the conditions that $\lambda_v^{i_1} \neq 0$ and $\sin(\gamma^{i_1}) \neq (q^{i_1} \cdot s)/(2K \cdot W)$:

$$\tan(\gamma^{i_1} - \gamma^o) = \frac{C_{D_0} \cdot (q^{i_1} \cdot s)^2 + K \cdot W^2 \cdot \cos^2(\gamma^{i_1}) + W \cdot q^{i_1} \cdot s \cdot \sin(\gamma^{i_1})}{W \cdot \cos(\gamma^{i_1}) \cdot [-2 \cdot K \cdot W \cdot \sin(\gamma^{i_1}) + q^{i_1} \cdot s]} \tag{13}$$

$$\lambda_v^{i_1} = \frac{-V^{i_1} \cdot \cos(\gamma^{i_1} - \gamma^o) \cdot q^{i_1} \cdot s}{g \cdot \sin(\gamma^o) \cdot \cos(\gamma^{i_1}) \cdot [2K \cdot W \cdot \sin(\gamma^{i_1}) - q^{i_1} \cdot s]} \tag{14}$$

C. Second Inner Boundary Layer

The second inner boundary layer is derived by changing the timescale from τ_1 to $\tau_2 = t/\varepsilon^2$ and then substituting $\varepsilon = 0$ into the general formalism. The slower variables are constant and equal to their initial values. The control of this layer is C_L. The only state variable is γ

$$\frac{\mathrm{d}\gamma^{i_2}}{\mathrm{d}\tau_2} = \left[\frac{L\left(C_L^{i_2}, q_0\right)}{W} - \cos(\gamma^{i_2})\right] \cdot \frac{g}{V_0} \tag{15}$$

The initial condition is γ_0, and the final condition matches the first inner-layer value

$$\gamma^{i_1}(t = 0)$$

The optimal control satisfies $\partial H/\partial C_L^{i_2} = 0$, which yields

$$C_L^{i_2} = \frac{1}{2K} \cdot \frac{\lambda_\gamma^{i_2}}{V_0 \cdot \lambda_{v_0}}$$

Because $\lambda_\gamma^{i_2}$ is proportional to $C_L^{i_2}$, one can replace it in equation $H = 0$ and obtain a second-order algebraic equation for the control, which produces two solutions:

$$C_{L_{(1,2)}}^{i_2} = C_L^{i_1}(\gamma^{i_2}) \mp \delta C_L(\gamma^{i_2}) \tag{16}$$

where

$$C_L^{i_1}(\gamma^{i_2}) = \frac{W \cdot \cos(\gamma^{i_2})}{q_0 \cdot s} \tag{17}$$

$$\left[\delta C_L(\lambda^{i_2})\right]^2 = \frac{C_{D_0}}{K} \cdot \left[1 - \frac{\sin(\gamma^{i_2} - \gamma^o)}{\sin\left(\gamma_0^{i_1} - \gamma^o\right)}\right]$$
$$+ \frac{W^2}{(q_0 \cdot s)^2} \cdot \left[\cos^2(\gamma^{i_2}) - \cos^2\left(\gamma_0^{i_1}\right) \cdot \frac{\sin(\gamma^{i_2} - \gamma^o)}{\sin\left(\gamma_0^{i_1} - \gamma^o\right)}\right]$$
$$+ \frac{W}{K \cdot q_0 \cdot s} \cdot \left[\sin(\gamma^{i_2}) - \sin\left(\gamma_0^{i_1}\right) \cdot \frac{\sin(\gamma^{i_2} - \gamma^o)}{\sin\left(\gamma_0^{i_1} - \gamma^o\right)}\right] \tag{18}$$

The sign ambiguity is resolved by taking the sign of $\gamma_0^{i_1} - \gamma^{i_2}$.

D. SPT Solution Construction

The initial control of the second boundary layer is a function of the initial state vector (x_0, h_0, V_0, γ_0). A feedback solution is obtained if the control of the second boundary layer is calculated at each step from the current state vector [$x(t)$, $h(t)$, $V(t)$, $\gamma(t)$], which represents the instantaneous initial conditions. An open-loop solution yields $C_L(t)$ and $\gamma(t)$. The state variables [$x(t)$, $h(t)$, $V(t)$] are derived by integrating their state equations with $C_L(t)$ and $\gamma(t)$.

V. Numerical Example

The following example compares the optimal trajectories of the SPT open loop and feedback laws to a numerical solution, which was obtained with the DNCONF optimization program of the IMSL software package. It solves a nonlinear programming problem using the successive programming algorithm and a finite difference gradient.

The air vehicle is defined by $W = 800$ lb, $s = 151$ ft^2, and $C_{D_0} = 0.01$, $K = 0.0216$. The resulting outer q^o is 7.78 lb/ft^2, and γ^o is -1.68 deg. The initial conditions are $h_0 = 1500$ ft, $V_0 = 400$ ft/s, $\gamma_0 = 0^0$, which represent an engine cutoff during medium subsonic cruise at sea level. The final condition is zero altitude. Figures 1–4 show the state variables and the control as function of time focusing on the boundary layer, and end at 120 s because the rest of the trajectory continues as a steady-state glide until the final altitude. In the second inner phase the open-loop and closed-loop control and state variables are similar and match the numerical results. In the first inner phase the open-loop path angle is still very close to the numerical one, whereas the closed-loop solution develops induced oscillations. However the average values of the oscillatory state variables are closer to the numerical solutions than the open-loop damped ones. Once the oscillations are damped, the variables in the closed-loop approximation coincide with the numerical ones.

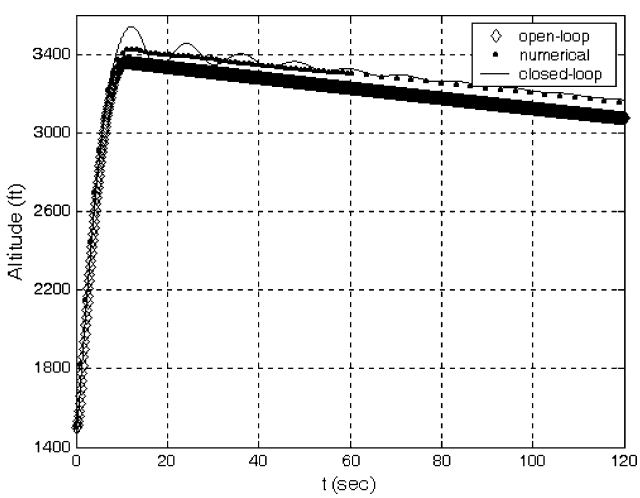

Fig. 1 Altitude vs time.

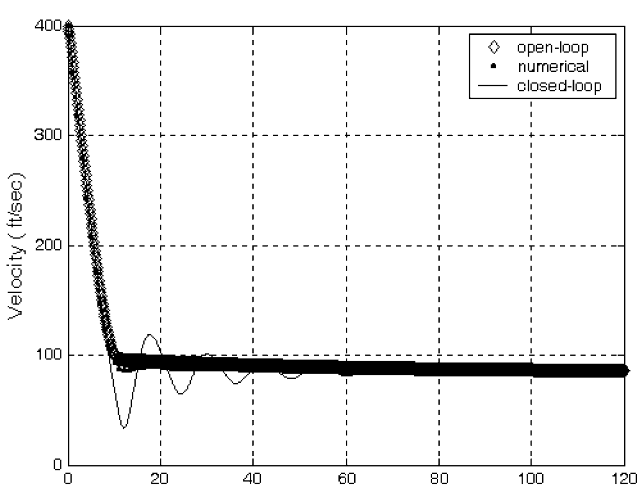

Fig. 2 Velocity vs time.

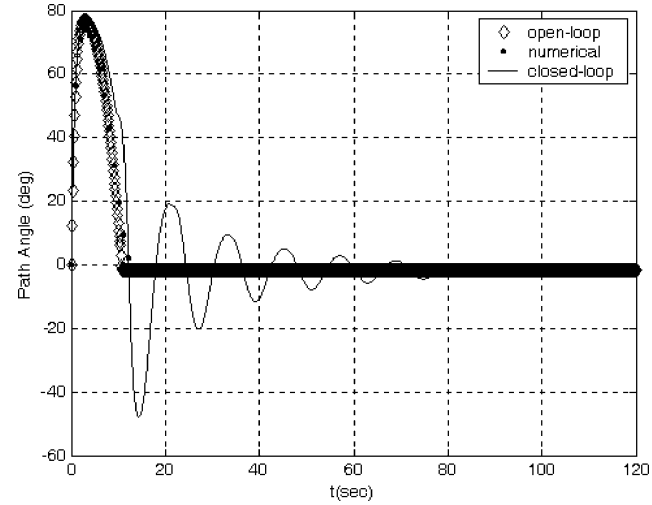

Fig. 3 Path angle vs time.

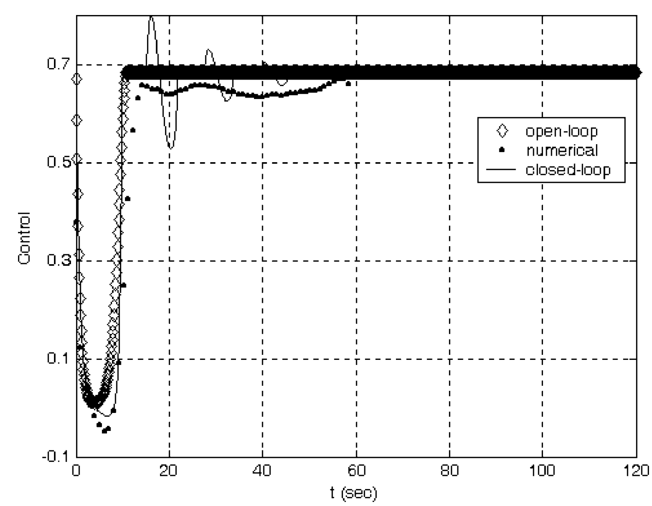

Fig. 4 Control vs time.

VI. Conclusions

Open-loop and closed-loop analytical trajectories for a glider were constructed and compared to a numerical solution. The open-loop solution shows good agreement with the numerical solution in all state variables, with minor range differences. The range difference stems from the fact that the altitude at the interception of the boundary layer with the outer section is slightly lower than the numerical one, and this altitude difference is translated into a range difference. The closed-loop solution produces range that coincides with the numerical optimal one, but is oscillatory in the control and all state variables in the intermediate phase. In spite of the oscillations, the range is very close to the numerical one, which indicates that the range is insensitive to the oscillations.

The oscillatory behavior indicates that the timescale separation, although good enough for the open-loop results, is not justified when the more sensitive closed-loop control is sought. The source of the closed-loop oscillations is probably the fact that the path angle does not change fast enough when approaching the outer section, so that the initial assumption of the timescale separation does not apply.

Both solutions are simple and easy to implement, whereas the numerical solutions rarely converge and are time consuming.

Analysis of the timescale separation is required in order to improve the SPT formulation of the problem and consequently the results.

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Modern Explicit Guidance Law for High-Order Dynamics

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I. Introduction

OPTIMAL control theory has been utilized to derive modern guidance laws with improved performance. This is an attempt to replace the well-known proportional navigation (PN). The requirement for better performance has led to the development of optimal guidance laws (OGLs) by the consideration of the target future maneuver and the dynamics of an interceptor and its target from perfect to time-invariant high-order autopilots.[1–4]

The desired performance index in the exoatmosphere is usually the amount of fuel required for corrective maneuvers. If the fuel consumption were minimized, the solution would be mathematically intractable,[1] especially with trajectory constraints. Explicit guidance laws (EGLs) may be developed to deal with this objective function. The original idea of the EGLs comes from Cherry, who developed a zero-miss guidance from a given time history of acceleration command assuming a vehicle with perfect dynamics.[5] Later, a zero-miss EGL was presented to improve autopilot lag compensation for arbitrary-order autopilots. Blackburn obtained a closed-loop terminal guidance for a PN-like acceleration profile for time-invariant autopilots.[6] This type of acceleration profile is not suitable for nonminimum phase autopilots.

Depending on specified mission, a midcourse strategy may be programmed.[7,8] Massoumnia developed an optimal midcourse strategy followed by a coasting phase for a perfect autopilot.[7] In this work, a nonzero-miss EGL is developed for an interceptor having a linear time-variant arbitrary-order autopilot. This approach is also extended for midcourse guidance strategies for two classes of systems. In addition, two classes of optimal midcourse guidance laws are obtained.[9,10] The proposed scheme, as well as the optimal control theory, assumes that the target future maneuvers are completely defined.

II. Explicit Guidance Problem and Solution

A linear time-varying system is represented in the following state-space form:

$$\dot{\boldsymbol{X}} = A(t)\boldsymbol{X} + B(t)\boldsymbol{U} + \boldsymbol{C}(t) \tag{1}$$

where $\boldsymbol{X}(\cdot)$ is the state vector, $\boldsymbol{U}(\cdot)$ the control vector, $A(t)$ the system matrix, $B(t)$ the input matrix, and $\boldsymbol{C}(t)$ a time-varying vector. All vectors and matrices are of appropriate dimensions. The final state at the final time t_f is given by

$$\boldsymbol{X}(t_f) = \boldsymbol{\Phi}(t_f, t)\boldsymbol{X}(t) + \int_t^{t_f} \boldsymbol{\Phi}(t_f, \lambda)B(\lambda)\boldsymbol{U}(\lambda)\,\mathrm{d}\lambda + \int_t^{t_f} \boldsymbol{\Phi}(t_f, \lambda)\boldsymbol{C}(\lambda)\,\mathrm{d}\lambda \tag{2}$$

with $\boldsymbol{\Phi}(t, t_0)$ being the fundamental matrix. We are to reach the desired state $\boldsymbol{X}^*(t_f)$ at the final time. The final deviation from the desired final state is denoted by $\boldsymbol{M} = \boldsymbol{X}^*(t_f) - \boldsymbol{X}(t_f)$. The predicted error at the final time without effort, that is, $\boldsymbol{U}(\xi) = \boldsymbol{0}$, $t < \xi \le t_f$, is expressed as

$$\boldsymbol{Z}(t) = \boldsymbol{X}^*(t_f) - \boldsymbol{\Phi}(t_f, t)\boldsymbol{X}(t) - \int_t^{t_f} \boldsymbol{\Phi}(t_f, \lambda)\boldsymbol{C}(\lambda)\,\mathrm{d}\lambda \tag{3}$$

Thus, we can write

$$\boldsymbol{M} = \boldsymbol{Z}(t) - \int_t^{t_f} \boldsymbol{\Phi}(t_f, \lambda)B(\lambda)\boldsymbol{U}(\lambda)\,\mathrm{d}\lambda \tag{4}$$

$$\boldsymbol{Z}(t) - \boldsymbol{Z}(t_0) = -\int_{t_0}^{t} \boldsymbol{\Phi}(t_f, \lambda)B(\lambda)\boldsymbol{U}(\lambda)\,\mathrm{d}\lambda \tag{5}$$

where the subscript 0 denotes the initial value. We are to construct a closed-loop guidance law from a given control history $\boldsymbol{U} = H(t)\boldsymbol{Z}(t_0)$, that is, H is given and its elements are piecewise continuous functions of time. Therefore, we have

$$\boldsymbol{Z}(t) = \left[I - \int_{t_0}^{t} \boldsymbol{\Phi}(t_f, \lambda)B(\lambda)H(\lambda)\,\mathrm{d}\lambda\right]\boldsymbol{Z}(t_0) \tag{6}$$

where I is the identity matrix. We assume that the expression in the brackets is invertible for $t_0 \le t \le t_f$. When the preceding relation is used, the closed-loop guidance law is obtained as

$$\boldsymbol{U} = H\left[\boldsymbol{W} + \int_t^{t_f} \boldsymbol{\Phi}(t_f, \lambda)B(\lambda)H(\lambda)\,\mathrm{d}\lambda\right]^{-1}\boldsymbol{Z}(t) \tag{7}$$

in which

$$\boldsymbol{W} = I - \int_{t_0}^{t_f} \boldsymbol{\Phi}(t_f, t)B(t)H(t)\,\mathrm{d}t \tag{8}$$

must satisfy the matrix equation $\boldsymbol{M} - \boldsymbol{W}\boldsymbol{Z}(t_0) = \boldsymbol{0}$ to reach the allowable predetermined final error $\boldsymbol{M}$. When the mentioned formulation is used, the EGL for final position and/or velocity constraints can be obtained for time-variant arbitrary-order autopilots. For this purpose, the elements of the guidance gain matrix corresponding to the states with unconstrained final values are set to zero.[9]

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